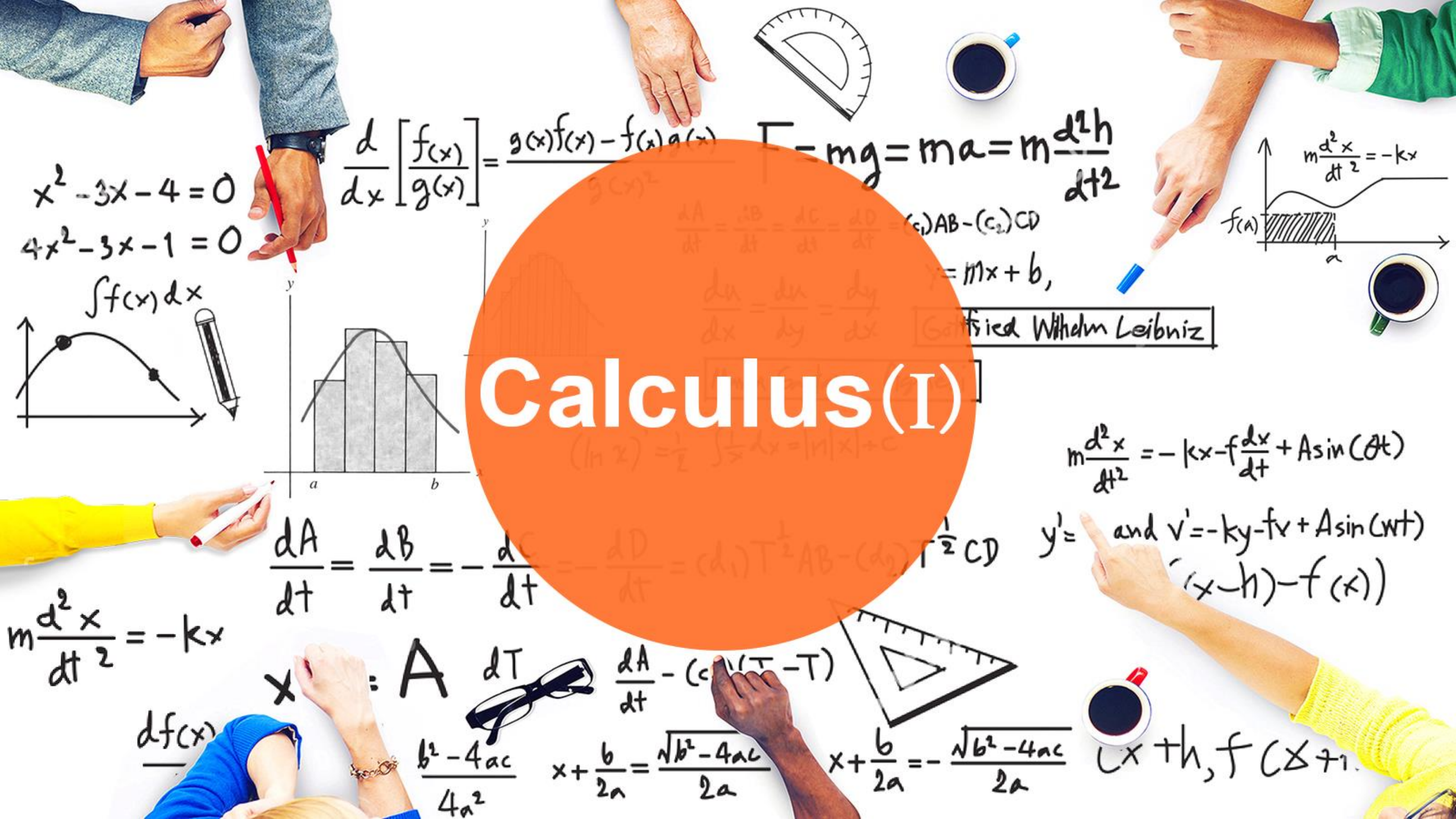
Calculus(Ⅰ)



$x^2-3x-4=0$
$4x^2-3x-1=0$
$\int f(x)\,dx$
$m\frac{d^2x}{dt^2}=-kx$
$F=mg=ma=m\frac{d^2h}{dt^2}$
$m\frac{d^2x}{dt^2}=-kx-f\frac{dx}{dt}+A\sin(\theta t)$
$x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$
$x+\frac{b}{2a}=-\frac{\sqrt{b^2-4ac}}{2a}$

Implicit Differentiation

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Problem Introduction

Def: $F(x,y)=0$ a function, the corresponding function $y=y(x)$ satisfying $F(x,y(x))\equiv 0$ is called implicit function.

Let $\sin y + xe^{y} = 0$, we cannot solve for y in terms of x,

In theory, $y=y(x)$ satisfying $\sin y(x) + xe^{y(x)} \equiv 0$,

How to find the derivation of y?

Implicit Differentiation

Derivation Rule of Implicit Function

Method: Derivation about x on both sides of the equation.

Tips: the variable y is a function of x.

The function of y is a composite function of x.

Use the composite chain rule for y.

Example 1

Find dy/dx if $x^2+5y^3=x+9.$

$$\frac{d}{dx}(x^2+5y^3)=\frac{d}{dx}(x+9)$$

$y=y(x)$

$$2x+15y^2\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1-2x}{15y^2}.$$

Example 2

$$\sin y + xe^{y} = 0, \text{ find } f'(x).$$

$$\cos y \cdot y'_x + 1 \cdot e^{y} + x e^{y} \cdot y'_x = 0$$

$y = y(x)$

$$\Rightarrow y'_x = \frac{-e^{y}}{\cos y + xe^{y}}.$$

Example 3

Find the equation of the tangent line to the curve

$x^3 + y^3 = 3xy$ at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.

$$3x^2 + 3y^2 \cdot y' = 3y + 3xy'$$

$$\therefore y'\Big|_{\left(\frac{3}{2},\frac{3}{2}\right)} = \frac{y - x^2}{y^2 - x}\Bigg|_{\left(\frac{3}{2},\frac{3}{2}\right)} = -1$$

The tangent line:

$$y - \frac{3}{2} = -(x - \frac{3}{2}) \Rightarrow x + y - 3 = 0.$$

Summary of Implicit Differentiation

Derivation Rule of Implicit Function

Method: Derivation about x on both sides of the equation.

The function of y is a composite function of x.

Use the composite chain rule for y.

Questions and Answers

Find the equation of the tangent line to the curve $y^3 - xy^2 + \cos xy = 2$ at the point (0, 1).

We differentiate both sides , so we have,

$$3y^2 y' - x(2yy') - y^2 - (\sin xy)(xy' + y) = 0$$

$$\Rightarrow y' = \frac{y^2 + y\sin xy}{3y^2 - 2xy - x\sin xy}$$

$$\Rightarrow y'\big|_{(0,1)} = \frac{1}{3} \quad \text{so} \quad y = \frac{1}{3}x + 1.$$

Questions and Answers

$x^4 - xy + y^4 = 1$, find the value of y'' at the point $(0,1)$

Method 1

$$4x^3 - y - xy' + 4y^3 \cdot y' = 0 \Rightarrow y'\Big|_{(0,1)} = \frac{1}{4}$$

Again derivation about x , so we have

$$12x^2 - y' - y' - xy'' + 12y^2 \cdot y' \cdot y' + 4y^3 \cdot y'' = 0$$

$$y''\Big|_{(0,1)} = -\frac{1}{16}.$$

Questions and Answers

$x^4 - xy + y^4 = 1$, find the value of y'' at the point $(0,1)$

Method 2

$$4x^3 - y - xy' + 4y^3 y' = 0$$

$$\Rightarrow y' = \frac{y - 4x^3}{4y^3 - x} \quad \Rightarrow y'\Big|_{(0,1)} = \frac{1}{4}$$

so we have, $y' = \dfrac{y - 4x^3}{4y^3 - x}$

$$y'' = \frac{(y' - 12x^2)(4y^3 - x) - (y - 4x^3)(12y^2 \cdot y' - 1)}{(4y^3 - x)^2}$$

$$\Rightarrow y''\Big|_{(0,1)} = -\frac{1}{16}.$$

Questions and Answers

Find the implicit equation of $xy - e^x + e^y = 0$, and find $y', y'\big|_{x=0}$

Let $y = y(x)$ replace y in the following equation,

then, $xy - e^x + e^y = 0$

derivation about x , so we have,

$$(xy)'_x - (e^x)'_x + (e^y)'_x = (0)'$$

$$y + xy' - e^x + e^y \cdot y' = 0 \Rightarrow y'\Big|_{\substack{x=0\\y=0}} = \frac{e^x - y}{e^y + x}\Big|_{\substack{x=0\\y=0}} = 1.$$

Implicit Differentiation